Comment on source finding using images of separate polarizations

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Middelberg et al. (2011) claim that processing right circular polarization (RCP) and left circular polarization (LCP) images separately and then identifying pair of peaks above the noise at both images at the same resolution element, reduces significantly the probability of a false detection of an unpolarised source with respect to the search of a peak at the image in Stokes I = (RCP + LCP)/2. They claim that in their case the probability of false detection is reduced by a factor of 6.6. This claim is counter-intuitive. This is equivalent to a claim that the signal to noise ratio can be reduced by splitting the set of *statistically independent data* into two subsets and processing them separately. In appendix A of Middelberg et al. (2011) their derivation of the probability of false detection is provided.

A close examination of that derivation reveals several errors that invalidate the claim. Firstly, authors claim that the number of random peaks, N', in an image above the threshold *is*

$$N' = Np, \tag{1}$$

where N is the number of independent resolution elements and p is the probability of having the brightness at a given resolution element exceeding the threshold $a\sigma$. This claim does not have sense. An image may have any number of peaks exceeding $a\sigma$, from 0 to N. The probability of image to have *exactly* N' peaks with the amplitude above the threshold is determined by a binomial distribution

$$P(N|N') = \binom{N}{N'} p^{N'} (1-p)^{N-N'}. \tag{2}$$

I guess the authors wanted to write that when the probability of brightness at a resolution element exceeding the threshold level $a\sigma$ is p, the mathematical expectation of the number of peaks above $a\sigma$ in the image of N resolution elements is Np.

Secondly, the authors write in the appendix A: "selecting one resolution element in RCP and N from the LCP image therefore results in $N' \times 1/N$ chance coincidences. Repeating that N times then results in N'^2/N chance coincidences." This is wrong. Unless p is too small, the probability that either the LCP or RCP map has 2, 3, 4, ... N peaks is not negligible. It is necessary to account not only for a case when the RCP map has N' peaks and the LCP map has only one peak, but also all combinations of $N', N'' \in [1, N]$ where N' is the number of peaks in the RCP map and N'' is the number of peaks in the LCP map.

Thirdly, the authors write "We assume that the noise in the images has a Gaussian distribution". According to their expression A.2, they also assume that the noise in images has zero mean. But the image is a brightness distribution, the energy per the unit surface area, per the unit frequency range, per the unit of time. It cannot be negative. An image in interferometry is produced by performing the inverse Fourier transform of visibility data. The thermal noise in complex visibility data is a complex Gaussian process with zero mean. Since the Fourier-transform is a linear operation, it does not change the shape of the distribution. The image, i.e. the amplitude of the Fourier-transform of the visibility data, has a Rayleigh distribution. Image processing algorithms, such as CLEAN, self-calibration, or the maximum entropy method, are non-linear transformations. However, if applied correctly, they should change only signal and not affect the noise. An over-cleaned image may have negative pixels, but these is an artifact of image processing and should be investigated separately.

The cumulative Rayleigh distribution determines the probability that at a given resolution element the amplitude exceeds $a\sigma$:

$$p(a) = e^{-a^2/2}. \tag{3}$$

The probability that a map with N resolution elements has at least one element with the amplitude exceeding $a\sigma$ is

$$p(a|N) = 1 - (1 - e^{-a^2/2})^N. \tag{4}$$

If we have two one-element "maps" at RCP and LCP, the probability that both elements have the amplitude exceeding $a\sigma$ is $p'(a) = p(a)^2 = e^{-a^2}$. Let reorganize RCP and LCP maps in one N-element map that is made of pairs of RCP/LCP resolution elements. The probability that an N-element RCP/LCP combined map has at least one pair with the amplitudes of both elements exceeding $a\sigma$ is

$$p'(a|N) = 1 - (1 - e^{-a^2})^N. \tag{5}$$

We see that $p'(a|N) = p(a/\sqrt{2}|N)$. Contrary to claim of Middelberg et al. (2011), the probability of false detection when LCP and RCP images are processed separately is *exactly the same* when a combined (RCP+LCP)/2 image is processed, provided the noise in images is statistically independent.

At the same time, source detection by a search of coinciding peak at LCP and RCP images *may* has a certain merit. If the polarization complex bandpass calibration is imperfect, the amplitude of the (LCP+RCP)/2 image of an unpolarised source may be less than the amplitude of LCP and RCP images. Although the probability of false detection in both cases remains the same, the chances of detection of a peak with a signal of a given amplitude at the (LCP+RCP)/2 image may be less because the signal is weaker. Second, due to a polarization leakage, the noise at RCP and LCP is not completely statistically independent, and therefore, the probability $p'(a|N)$ may be slightly different from expression 5.

References

Middelberg, E., Deller, A., Morgan, J., Rottmann, H., Alef, W., Tingay, S., Norris, R., Bach, U., Brisken, W., Lenc, E., Wide-field VLBA observations of the Chandra deep field South, 2011, Astron. & Astrophys., 526, A74